SIGNATURE ____________________ NAME ______________________

Student ID # ____________________

Physics 410
Fall 2015
Prof. Anlage
Exam 2
19 November, 2015
CLOSED BOOK, No Phones, Calculator Permitted

Point totals are given for each part of the question.
If you run out of room, continue writing on the back of the same page. If you do so, make a note on the front part of the page!
Note: You must solve the problem following the instructions given in the problem. Correct answers alone will not receive full credit.

Partial Credit:

→ Show Your Work! Answers written with no explanation will not receive full credit.

→ You can receive credit for describing the method you would use to solve a problem, even if you missed an earlier part.

Problem	Credit	Max. Credit
1		25
2		25
3		25
4		25
TOTAL		100

$\vec{r}\cdot\vec{s} = rs\cos\theta$ $\quad \vec{r}\times\vec{s} = det\begin{bmatrix}\hat{x} & \hat{y} & \hat{z}\\ r_x & r_y & r_z\\ s_x & s_y & s_z\end{bmatrix}$ $\quad \vec{F} = m\ddot{\vec{r}}$ $\quad Constant\ a\colon x(t) = x_0 + v_0 t + \frac{1}{2}at^2; v(t) = v_0 + at;\ v_f^2 - v_i^2 = 2a\Delta x$ $\quad \vec{f} = -f(v)\hat{v}$ $\quad f(v) = bv + cv^2 = \beta D v + \gamma D^2 v^2$

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$ $\quad m\dot{v} = -\dot{m}v_{ex} + F^{ext}$ $\quad v - v_0 = v_{ex}\ln\frac{m_0}{m}$ $\quad \vec{R} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha$

$\vec{R} = \frac{1}{M}\int \vec{r}\,dm = \frac{1}{M}\int \rho\,\vec{r}\,dV$ $\quad \vec{\ell} = \vec{r}\times\vec{p}$ $\quad \vec{L} = \sum_{\alpha=1}^{N}\vec{r}_\alpha\times\vec{p}_\alpha$ $\quad \dot{\vec{L}} = \vec{\Gamma}^{ext}$ $\quad I = \sum_\propto m_\propto \rho_\alpha^2$ $\quad \Delta T = T_2 - T_1 = \int_1^2 \vec{F}\cdot d\vec{r} = W(1\to 2)$ $\quad T = mv^2/2$ $\quad U(\vec{r}) = -W(\vec{r}_0\to\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r'})\cdot d\vec{r'}$ $\quad \vec{\nabla}\times\vec{F} = 0$ $\quad \vec{F} = -\vec{\nabla}U$ $\quad E = T + U_1 + \cdots + U_n$ $\quad \Delta E = W_{nc}$

$t = \sqrt{\frac{m}{2}}\int_{x_0}^{x}\frac{dx\prime}{\sqrt{E - U(x')}}$ $\quad \vec{F}(\vec{r}) = f(\vec{r})\hat{r}$ $\quad U = U^{int} + U^{ext} = \sum_\alpha\sum_{\beta>\alpha} U_{\alpha\beta} + \sum_\alpha U_\alpha^{ext}$

$Net\ force\ on\ particle\ \alpha = -\nabla_\alpha U$ $\quad T + U = constant$ $\quad F = -kx \leftrightarrow U = \frac{1}{2}kx^2$

$\ddot{x} = -\omega^2 x \leftrightarrow x(t) = A\cos(\omega t - \delta)$ $\quad \ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \leftrightarrow x(t) = Ae^{-\beta t}\cos(\omega_1 t - \delta)\ (assuming\ \beta < \omega_0), \beta = \frac{2b}{m}, damping\ force = -bv, \omega_0 = \sqrt{\frac{k}{m}}, \omega_1 = \sqrt{\omega_0^2 - \beta^2}$

$F(t) = mf_0\cos(\omega t), x(t) = A\cos(\omega t - \delta), where\ A^2 = \frac{f_0^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}$ $\quad \delta = \tan^{-1}\left(\frac{2\beta\omega}{\omega_0^2 - \omega^2}\right)$ $\quad S = \int_{x_1}^{x_2} f[y(x), y'(x), x]dx$, $\quad \frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$

$S = \int_{u_1}^{u_2} f[x(u), y(u), x'(u), y'(u), u]du$, $\frac{\partial f}{\partial x} = \frac{d}{du}\frac{\partial f}{\partial x'}, and\ \frac{\partial f}{\partial y} = \frac{d}{du}\frac{\partial f}{\partial y'}$ $\quad \mathcal{L} = T - U$

$\frac{\partial\mathcal{L}}{\partial q_i} = \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_i}\ [i = 1, \dots n]$ $\quad p_i = \frac{\partial\mathcal{L}}{\partial\dot{q}_i}$ $\quad \mathcal{H} = \sum_{i=1}^{n} p_i\dot{q}_i - \mathcal{L}$ $\quad \vec{r} = \vec{r}_1 - \vec{r}_2$ $\quad \mu = \frac{m_1 m_2}{m_1 + m_2}$

$U_{eff} = U(r) + U_{cf}(r) = U(r) + \frac{\ell^2}{2\mu r^2}$ $\quad T = \frac{1}{2}M\dot{\vec{R}}^2 + \frac{1}{2}\mu\dot{\vec{r}}^2$ $\quad u''(\varphi) = -u(\varphi) - \frac{\mu}{\ell^2 u(\varphi)^2}F$

$r(\varphi) = \frac{c}{1 + \epsilon\cos\varphi}\ for\ F = -\frac{\gamma}{r^2}, with\ c = \frac{\ell^2}{\gamma\mu}$ $\quad E = \frac{\gamma^2\mu}{2\ell^2}(\epsilon^2 - 1)$

$\vec{F}_{inertial} = -m\vec{A}$ $\quad \vec{\omega} = \omega\hat{u}$ $\quad \vec{v} = \vec{\omega}\times\vec{r}$ $\quad \left(\frac{d\vec{Q}}{dt}\right)_{S_0} = \left(\frac{d\vec{Q}}{dt}\right)_S + \vec{\Omega}\times\vec{Q}$

$m\ddot{\vec{r}} = \vec{F} + \vec{F}_{cor} + \vec{F}_{cf}, with\ \vec{F}_{cor} = 2m\dot{\vec{r}}\times\vec{\Omega}, and\ \vec{F}_{cf} = m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$

$\vec{g} = \vec{g}_0 + (\vec{\Omega}\times\vec{R})\times\vec{\Omega}$ $\quad N_{oc} = N_{inc}n_{tar}\sigma_{oc}$ $\quad N_{sc}(into\ d\Omega) = N_{inc}n_{tar}\frac{d\sigma}{d\Omega}(\theta,\phi)d\Omega$

$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$ $\quad \frac{d\sigma}{d\Omega} = \left(\frac{kqQ}{4\mathrm{E}\sin^2(\theta/2)}\right)^2$ $\quad \dot{q}_i = \partial\mathcal{H}/\partial p_i$ and $\dot{p}_i = -\frac{\partial\mathcal{H}}{\partial q_i}\ [i = 1, \dots n]$

$\vec{L} = \vec{L}(motion\ of\ CM) + \vec{L}(motion\ relative\ to\ CM)$

$T = T(motion\ of\ CM) + T(motion\ relative\ to\ CM)$ $\quad \vec{L} = \bar{\bar{I}}\vec{\omega}$

$\bar{\bar{I}} = \begin{bmatrix}I_{xx} & I_{xy} & I_{xz}\\ I_{yx} & I_{yy} & I_{yz}\\ I_{zx} & I_{zy} & I_{zz}\end{bmatrix}$ $\quad I_{xx} = \sum_\alpha m_\alpha(y_\alpha^2 + z_\alpha^2), etc.$ $\quad I_{xy} = -\sum_\alpha m_\alpha x_\alpha y_\alpha, etc.$ $\quad \vec{L} = \lambda\vec{\omega}$

$(\bar{\bar{I}} - \lambda\bar{\bar{1}})\vec{\omega} = 0$ $\quad$ Characteristic equation: $det(\bar{\bar{I}} - \lambda\bar{\bar{1}}) = 0$ $\quad \bar{\bar{I}}' = \begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{bmatrix}$ $\quad \dot{\vec{L}} + \vec{\omega}\times\vec{L} = \vec{\Gamma}$ $\quad \bar{\bar{M}}\ddot{\vec{q}} = -\bar{\bar{K}}\vec{q}$ $\quad T = \frac{1}{2}\sum_{j,k} M_{jk}\,\dot{q}_j\dot{q}_k$ $\quad U = \frac{1}{2}\sum_{j,k} K_{jk}\,q_j q_k$ $\quad \vec{q}(t) = Re(\vec{a}\,e^{i\omega t})$

$\left(\bar{\bar{K}} - \omega^2 \bar{\bar{M}}\right)\vec{a} = 0$ $\ddot{\phi} + 2\beta\dot{\phi} + \omega_0^2 \sin\phi = \gamma\omega_0^2 \cos(\omega t)\,, with\ \gamma = \frac{F_0}{mg}, and\ F(t) = F_0 \cos(\omega t)$ $\dot{q}_i = \partial\mathcal{H}/\partial p_i$ and $\dot{p}_i = -\frac{\partial\mathcal{H}}{\partial q_i}$ $[i = 1, \dots n]$

$G(t,t') = \begin{cases} \frac{e^{-\beta(t-t')}\sin\left(\omega_1(t-t')\right)}{m\omega_1} \text{ for } t \geq t' \\ 0 \text{ for } t < t' \end{cases}$ $x(t) = \int_{-\infty}^{t} F(t')\, G(t,t') dt'$

Binomial expansion for x <<1: $(1+x)^n \cong 1 + nx + \frac{n(n-1)}{2}x^2$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots \quad (x < 1)$$